

KIX 1001: ENGINEERING MATHEMATICS 1

Tutorial 6: Matrix Algebra

1. Let $A = \begin{bmatrix} 5 & -2 & 0 \\ -2 & 6 & 2 \\ 0 & 2 & 7 \end{bmatrix}$, $X = \begin{bmatrix} -x \\ 0 \\ x \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -1 \\ 2 & -1 & 2 \\ -1 & 2 & 2 \end{bmatrix}$

(a) Find the value of x such that $X^T A X = 144$

(b) Show that $B^T A B = 27 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

2. Let

$$A = \begin{bmatrix} 6 & -1 & 1 \\ 0 & 13 & -16 \\ 0 & 8 & -11 \end{bmatrix} \quad \text{and} \quad x = \begin{bmatrix} 10.5 \\ 21.0 \\ 10.5 \end{bmatrix}$$

(a) Determine a scalar r such that $Ax = rx$

(b) Is it true $A^T x = rx$ for the value of r determined in part (a)

3. Solve each of the following systems of linear equations using Gaussian Elimination technique

$$x + 2y + 3z = 9$$

(a) $2x - y + z = 8$

$$3x - z = 3$$

$$-3x + 2y - 6z = 6$$

(b) $5x + 7y - 5z = 6$

$$x + 4y - 2z = 8$$

$$2x + y + 3z = 1$$

(c) $2x + 6y + 8z = 3$

$$6x + 8y + 18z = 5$$

$$2x - 3y + z = -5$$

(d) $3x + 2y - z = 7$

$$x + 4y - 5z = 3$$

$$x + y + z = 6$$

(e) $2x - y + z = 3$

$$3x - z = 0$$

4. Find the eigenvalues and their associated eigenvectors

(a) $A = \begin{bmatrix} 7 & 0 & -3 \\ -9 & -2 & 3 \\ 18 & 0 & -8 \end{bmatrix}$

(b) $A = \begin{bmatrix} -5 & 0 & 0 \\ 3 & 7 & 0 \\ 4 & -2 & 3 \end{bmatrix}$

5. Using Cayley Hamilton approach, find A^{-1} for the following matrix:

(a) $\begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$

6. Diagonalize the following matrix, if possible

(a) $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 4 & 6 \\ 0 & 2 & 2 \\ 0 & 0 & 4 \end{bmatrix}$