KIX 1001: ENGINEERING MATHEMATICS 1

Tutorial 5: Vector Algebra I

1. Two long straight pipes are specified using Cartesian coordinates as follow:
Pipe A: diameter 0.8; axis through points (2,5,3) and (7,10,8).
Pipe B: diameter 1.0; axis through points (0,6,3) and (-12,0,9).
Do the pipes need re-aligning to avoid intersection?

2. Find the distance from $P = (-3,7,4)$ to the line l with vector equation $\boldsymbol{r} = \begin{pmatrix} 2 \\ -2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix}$.

3. Calculate the distance between the lines l and m having vector equations $\boldsymbol{r} = \boldsymbol{a} + \lambda \boldsymbol{u}$ and $\boldsymbol{r} = \boldsymbol{b} + \mu \boldsymbol{v}$ respectively, where
$\boldsymbol{a} = \begin{pmatrix} 0 \\ 4 \\ -1 \end{pmatrix}$, $\boldsymbol{u} = \begin{pmatrix} 1 \\ -3 \\ -2 \end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}$.

4. Find the Cartesian equation of plane contains the point (1,2,-1) and perpendicular to the intersecting line of the planes and $2x + y + z = 2$ and $x + 2y + z = 3$.

5. Find the Cartesian equation of plane contains the line L_1: $\boldsymbol{r_1} = \boldsymbol{a} + t\boldsymbol{u} = \langle 1, -3,4 \rangle + \langle 2,1,1 \rangle t$ and parallel to the line L_2: $\boldsymbol{r_2} = \boldsymbol{b} + s\boldsymbol{v} = \langle 0,0,0 \rangle + \langle 1,2,3 \rangle s$. From the result, can you proof that the plane is parallel to line L_2?

6. Find the Cartesian equation of plane contains the line L_1: $\boldsymbol{r_1} = \boldsymbol{a} + t\boldsymbol{u} = \langle -2,3,4 \rangle + \langle 1,2,-1 \rangle t$ and line L_2: $\boldsymbol{r_2} = \boldsymbol{b} + s\boldsymbol{v} = \langle 3,4,0 \rangle + \langle -1,-2,1 \rangle s$.

7. Find the unit vectors that are perpendicular to the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ as following:
i. $\underset{\sim}{a} = \langle 2,4,5 \rangle$, $\underset{\sim}{b} = \langle 1,2,-2 \rangle$
ii. $\underset{\sim}{a} = \langle 2,4,-4 \rangle$, $\underset{\sim}{b} = \langle 1,2,-2 \rangle$

8. Let $\underset{\sim}{a} = \langle 1,-2,-3 \rangle$, $\underset{\sim}{b} = \langle 2,1,-1 \rangle$ and $\underset{\sim}{c} = \langle 1,3,-2 \rangle$. Find
i. $\underset{\sim}{a}.\underset{\sim}{b}(\underset{\sim}{a} \times \underset{\sim}{b})$
ii. $(\underset{\sim}{a} + \underset{\sim}{b}) \times \underset{\sim}{c}$