KIX 1001: ENGINEERING MATHEMATICS 1

Tutorial 3: Partial Derivatives & Engineering Applications of Partial Derivatives

1. Find the partial derivatives $\left(\frac{\partial f}{\partial y}\right)$ and $\left(\frac{\partial f}{\partial x}\right)$of these functions using the limit definition
   (a) $f(x,y) = x^2 - 4xy + y^2$
   (b) $f(x,y) = 2x^3 + 3xy - y^2$

2. Determine all the first and second order partial derivatives of the function
   (a) $f(x,y) = x^2y^3 + 3y + x$
   (b) $f(x,y) = y \sin x + x \cos y$
   (c) $f(x,y) = x^4 \sin 3y$
   (d) $f(x,y) = e^{xy} (2x – y)$
   (e) $f(x,y,z) = z^2e^{xy} + x \cos (y^2z)$

3. Find both partial derivatives for each of the following two variables functions
   (a) $f(x,y) = \log x + 3y + 1$
   (b) $g(x,y) = ye^{x+y}$
   (c) $h(x,y) = xsiny - ycos\ x$
   (d) $p(x,y) = x^y + y^2$
   (e) $U(x,y) = \frac{9y^3}{x-y}$

4. Compute the first and second partial derivatives of z

$$z = \frac{x}{y^2} + \frac{1}{x^3} + \log(x + y)$$

5. For f(x,y,z), use the implicit function theorem to find dy/dx and dy/dz:
   (a) $f(x,y,z) = x^2y^3 + z^2 + xyz$
   (b) $f(x,y,z) = x^3z^2 + y^3 + 4xyz$
   (c) $f(x,y,z) = 3x^2y^3 + xz^2y^2 + y^3zx^4 + y^2z$

6. Find $\frac{\partial F}{\partial s}$ and $\frac{\partial F}{\partial t}$,if applicable, for the following composite functions
   (a) $F = \sin (x + y)$ where $x = 2st$ and $y = s^2 + t^2$
   (b) $F = e^x \cos y$ where $x = s^2 – t^2$ and $y = 2st$ (book exercise 9.2.6)
   (c) $F = 5x – 3y^2 + 7z^3$ where $x = 2s + 3t$, $y = s – t$ and $z = 4s + t$
   (d) $F = \ln (x^2 + y)$ where $x = \exp (s+t^2)$ and $y = s^2 + t$
   (e) $F = x^2y^2$ where $x = s \cos t$ and $y = s \sin t$
   (f) $F = xy + yz^2$ where $x = e^t$, $y = e^t \sin t$ and $z = e^t \cos t$

7. Find dy/dx and dy/dz (if applicable) for each of the following
   (a) $y^x + 1 = 0$

(b) $7x^2 + 2xy^2 + 9y^4 = 0$
(c) $x^3z^2 + y^3 + 4xyz = 0$
(d) $3x^2y^3 + xz^2y^2 + y^3zx^4 + y^2z = 0$
(e) $y^5 + x^2y^3 = 1 + y \exp(x^2)$

8. (a) Show that the variables x and y given by $x = \frac{s+t}{s}, y = \frac{s+t}{t}$ are functionally dependent.
   (b) Obtain the Jacobian J of the transformation s = 2x + y, t = x – 2y and determine the inverse of the transformation J_1. Confirm that $J_1=J^{-1}$.
   (c) Show that if x + y = u and y = uv, then $\frac{\partial(x,y)}{\partial(u,v)} = u.$
   (d) Verify whether the functions $u = \frac{x+y}{1-xy}$ and v = $\tan^{-1}x + \tan^{-1}y$ are functionally dependent.
   (e) If x = uv, $= \frac{u+v}{u-v}$, find $\frac{\partial(u,v)}{\partial(x,y)}$.

Total differential

1. Compute the total differential of f(x,y,z) = ln $(\frac{xy^2}{z^3})$.
2. The period T of a simple pendulum is T = $2\pi\sqrt{\frac{l}{g}}$, find the maximum percentage error in T due to possible errors up to 1% in *l* and 2% in g (Hint: $\frac{dl}{l}$ = 0.001 and $\frac{dg}{g}$ = 0.002)
3. Compute an approximate value of $(1.04)^{3.01}$
4. Suppose one is given a triangle where the angle at one vertex is θ and the lengths of the two sides adjacent to that vertex are *b* and *c*. Then the well-known formula for the area of the triangle as

$$Area = S = \frac{1}{2} bc \sin\theta$$

   Suppose that we measure:

$$b = 4.00\,m \pm 0.005m$$
$$c = 3.00\,m \pm 0.005m$$
$$\theta = \frac{\pi}{6} \pm 0.01\,radians$$

   Find *S* and estimate the percentage error.

Tangent planes and normal to surfaces in three dimensions

Find the equations of the tangent plane and normal line to the following surfaces at the points indicated:

1. $x^2 + 2y^2 + 3z^2 = 6$ at (1,1,1)
2. $2x^2 + y^2 - z^2 = -3$ at (1,2,3)
3. $x^2 + y^2 - z = 1$ at (1,2,4)
4. $\ln\left(\frac{x}{y}\right) - z^2(x - 2y) - 3z = 3$ at (4,2,-1)
5. $x^3z + z^3x - 2yz = 0$ at (1,1,1)

6. $z = 5+(x-1)^2 + (y+2)^2$ at (2,0,10)
7. $\frac{x^2}{12} + \frac{y^2}{6} + \frac{z^2}{4} = 1$ at (1,2,1)
8. $ze^x+e^{z+1}+xy+y=3$ at (0,3-1)
9. $z^2 = 7 - x^2 - 2y^2$ at (1,1,2)
10. $z = \frac{x^2}{2p} + \frac{y^2}{2q}$ at the point $(2p^2, 2q^2, 2p^3+2q^3)$