KIX 1001: ENGINEERING MATHEMATICS 1 (2018/19)

Tutorial 13: Power Series Solutions

1. Find the radius of convergence and interval of convergence for the given power series.

   a. $\sum_{n=1}^{\infty} \frac{2^n}{n} x^n$ b. $\sum_{n=1}^{\infty} \frac{(-1)^n}{4^n} (x+3)^n$

   c. $\sum_{n=0}^{\infty} \frac{(100)^n}{n!} (x+7)^n$ d. $\sum_{n=0}^{\infty} n!\,(2x+1)^n$

2. Rewrite the given power series by shifting the index, so that its general term involves x^k.

   a. $\sum_{n=3}^{\infty} (2n-1) c_n x^{n-3}$ b. $\sum_{n=3}^{\infty} \frac{3^n}{(2n)!} x^{n-2}$

   c. $\sum_{n=3}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$

3. Rewrite the given expression as a single power series whose general term involves x^k.

   a. $\sum_{n=2}^{\infty} n(n-1) c_n x^n \;+\; 2\sum_{n=2}^{\infty} n(n-1) c_n x^{n-2} + 3\sum_{n=1}^{\infty} n c_n x^n$

   b. $3x^2 \sum_{n=-2}^{\infty} n(n-1) x^{n-2} + x \sum_{n=1}^{\infty} n x^n$

   c. $\sum_{n=1}^{\infty} \frac{3^n}{(2n)!} x^{n-1} \;+\; 2x^3 \sum_{n=-1}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$

4. Find two power series solutions of given differential equation about the ordinary point x =0.

   a. $y'' + xy' + y = 0$

   b. $(x-1\,)y'' + y' = 0$

   c. $y'' + e^x y' - y = 0$

   d. $(x^2+1)y'' + xy' - y = 0$

5. Use the power series method to solve the given initial-value problem.

a. $y'' - xy' - y = 0, \quad y(0) = 1, \ y'(0) = 0$

b. $y'' + x^2y' + xy = 0, \quad y(0) = 0, \ y'(0) = 1$

c. $(x+1)\,y'' - (2-x)y' + y = 0, \quad y(0) = 2, \ y'(0) = -1$

6. Determine the singular points of the given differential equation. Clasify each singular point as regular or irregular.

a. $x^3y'' + 4x^2y' + 3y = 0$

b. $(x^2-9)^2y'' + (x+3)y' + 2y = 0$

c. $(2x^2 - 5x - 3)y'' + (2x+1)y' + \dfrac{6}{(x-3)}y = 0$

d. $(x^3 - 2x^2 - 3x)^2y'' + x(x-3)^2y' - (x+1)y = 0$

7. Find the indicial roots for the given differential equations where x = 0 is a regular singular point.

a. $2xy'' - y' + 2y = 0$

b. $3xy'' + (2-x)y' - y = 0$

c. $9x^2y'' + 9x^2y' + 2y = 0$

d. $x^2y'' + xy' + \left(x^2 - \frac{4}{9}\right)y = 0$

e. $xy'' + (1-x)y' - y = 0$