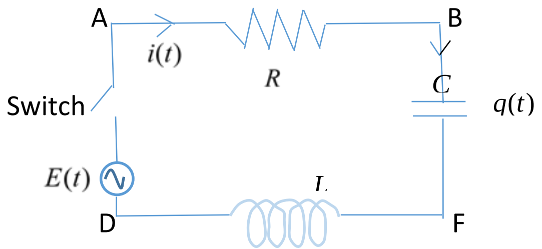
KIX 1001: ENGINEERING MATHEMATICS 1
Tutorial 12: 2^{nd} Order Differential Equation

1. Given the governing equation for RLC electrical circuit: $L\frac{d^2q(t)}{dt^2}+R\frac{dq(t)}{dt}+\frac{1}{C}q(t)=E(t)$.

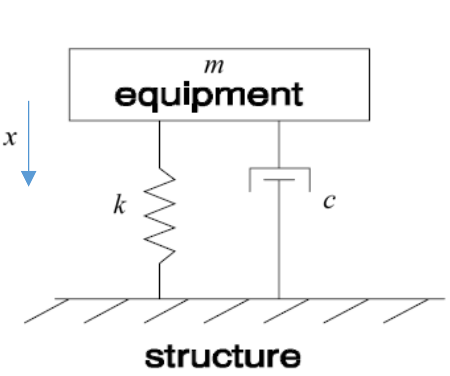
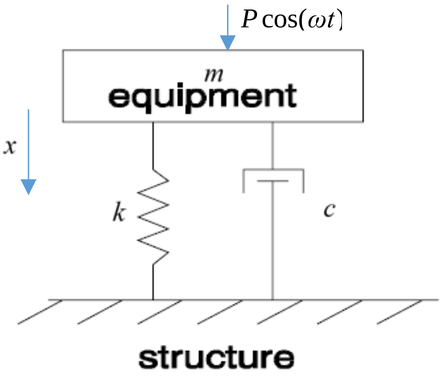
   An inductor of $L=$ 50 henrys, a resistor of $R=$ 5 ohms and a capacitor of $C=8$ farads are connected in series with an emf of E volts. At $t=0$, the switch S is closed. Find the charge and current at any time $t>0$ if the voltage is supplied by (a) DC battery, $E(t)=\boldsymbol{a}$ volts or (b) AC generator, $E(t)=\boldsymbol{b}e^{-3t}$ volts.

   Replace $\boldsymbol{a}$ with the last three digits of your matric number. For example, if your matric number is KHA110108, your $\boldsymbol{a}$ is thus 108.

   Replace $\boldsymbol{b}$ with the last three digits of your matric number divided by 5. For example, if your matric number is KHA110108, your $\boldsymbol{b}$ is thus 108/5.



2. The vibration transmission from the effect of equipment/ machine vibration to its structure (e.g. washing machine attached to the ground or engine attached to the car structure) can be modelled as 1 DOF spring-damper-mass vibration problem. It can be categorised into two conditions as follows.

<table>
<tr><th>(a) Transient Condition
(Free Vibration)</th><th>(b) Steady State Condition
(Forced Vibration)</th></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>Governing equation:

$m\frac{d^2x(t)}{dt^2} + c\frac{dx(t)}{dt} + kx(t) = 0$ (machine at rest)

$x(0) = x_0, \dot{x}(0) = \dot{x_0}$</td><td>Governing equation:

$m\frac{d^2x(t)}{dt^2} + c\frac{dx(t)}{dt} + kx(t) = P\cos\omega t$
(machine is rotating with cyclic/harmonic force)

$x(0) = x_0, \dot{x}(0) = \dot{x_0}$</td></tr>
<tr><td>The homogeneous 2nd order ODE is known as characteristic equation because it represents the chactacteristic of many systems. It has the complementary solution (y_c).</td><td>The solution of nonhomogeneous 2nd order ODE is the summation of complementary solution (y_c) and particular solution (y_p).</td></tr>
</table>

Note: In vibration field, the characteristic eqn. determines the dynamic characteristic of the vibrating system such as the natural frequency which causes mechanical resonance phenomenon. By understanding the dynamic behaviour of the system though the formulation of ODE, engineer can design a safer and reliable structure/ machine. In electrical field, engineers utilize the electrical resonance in radio tuning application through the formulation of ODE's characteristic eqn. The detail of these are out of the scope in this study. Students are encouraged to utilize the basic of the mathematical tool learned in this course for their future engineering application.

2. Let the governing equation for a vibrating car structure:

$2\frac{d^2x(t)}{dt^2} + 7\frac{dx(t)}{dt} + 8x(t) = F(t)$; where $F(t)$ is the forcing function and $x(0) = 2, \dot{x}(0) = 0$.

Find the total solution for the 2nd order ODE equation if the forcing function is given as follows:

(a) No excitation, $F(t) = 0$ and it is subjected to initial condition.

(b) Repeat the same problem in 2(a) with various combinations of damping, i.e. $2\frac{d^2x(t)}{dt^2} + 8\frac{dx(t)}{dt} + 8x(t) = F(t)$.

(c) Repeat the same problem in 2(a) with various combinations of damping, i.e. $2\frac{d^2x(t)}{dt^2} + 9\frac{dx(t)}{dt} + 8x(t) = F(t)$.

3. Continue the problem 2. Let the governing equation for a vibrating car structure: $2\frac{d^2x(t)}{dt^2} + 7\frac{dx(t)}{dt} + 8x(t) = F(t);$ where $F(t)$ is the forcing function and $x(0) = 2, \dot{x}(0) = 0$. Find the total solution for the 2nd order ODE equation if the forcing function is given as follows:

(a) Engine excitation $F(t) = 5\cos 10t$
(b) Engine excitation $F(t) = 8\sin 8t$ $F(t) = 8\sin 8t$
(c) Engine excitation $F(t) = e^{-10t}$
(d) Engine excitation $F(t) = e^{-10t}\cos 10t$ [Hint / Alternative: Superposition]
(e) Engine excitation $F(t) = 5\cos 10t + e^{-10t}$
(f) Road excitation $F(t) = 10$
(g) Road excitation $F(t) = 5t^2 + 7\mathrm{t} + 9$
(h) Road excitation $F(t) = 6te^t + 3t$

Hint: Student just need to show an example for the solution of homogenous part once and do not need to repeat the same step in other examples if it is needed. To further master the skill to solve 2nd order ODE problem, students can repeat Q3(a-h) for various combinations of damping as shown in Q2(b) and Q2(c) respectively.