KIX 1001: ENGINEERING MATHEMATICS 1

Tutorial 11: Differential Equations (First Order)

1. Identify 5 physical laws/ theory that are frequently used in your field of study (i.e. Mechanical/Electrical/Chemical/Civil/Environment/etc.) and show that they can be transformed into the form of differential equation.

2. Identify the dependent & independent variables for each case. Classify each equation according to its order, linearity/non-linearity, and homogeneity/non-homogeneity. Hence, find its' solutions if it can be solved by using linear differential equation and separable differential equation.

   (i) $5x\frac{d^2y}{dx^2} - \frac{4}{x}\frac{dy}{dx} - \sin 2x = 0$, $y(0) = 0$, $y'(0) = 0$

   (ii) $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} = 4y$, $y(0) = 0$, $y'(0) = 0$

   (iii) $\frac{dy}{dx} + \frac{4}{x}y - x^3y^2 = 0$, $y(2) = -1$

   (iv) $e^{-t^3}\frac{dx}{dt} - 3t^2e^{-t^3}x = 0$, $x(1) = 2$

   (v) $\frac{dv}{dt} - 3t^2v + 3t^2 = 0$, $v(0) = 2$

   (vi) $\frac{dy}{dt} - 3ty = \frac{t^2y^3}{y+1}$, $y(0) = 0$

   (vii) $x^2\frac{d^2y}{dx^2} - 3y\frac{dy}{dx} = 0$, $y(0) = 1$, $y(2) = 4$

   (viii) $x\frac{dy}{dx} + y = 8$, $y(3) = 5$

   (ix) $\frac{dy}{dt} = \frac{y^2 + yt}{t^2}$, $y(1) = 4$

   (x) $5x\frac{d^3y}{dx^3} - \frac{4}{x}\frac{dy}{dx} - 5\tan x = 0$, $y(0) = 0$, $y(5) = 4$, $y(10) = 7$

   (xi) $x'' + \sin(x) = 0$

   (xii) $x'' + 2x' + x = \sin(t)$

3. Find the solution of the differential equation $\frac{dy}{dx} = -2x$.
   a. Sketch the curves of the solution when the constant value is 1, 0 and -1, respectively.
   b. What would be the particular solution if given a boundary condition of $y(1) = 3$?

4. (a) Given the population of rabbit in human habitat (rabbit farm) grows at a rate proportional to the number of rabbit at time t (year). It is observed that 200 and 800 rabbits are presented

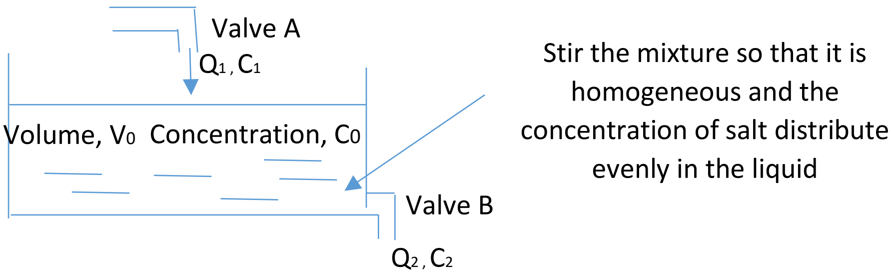
at 3rd year and 6th year respectively. What was the initial number of the rabbit, $y(0) = y_0$? How long does it take the population to double to $2y_0$?

(b) A person has bought 1000 rabbits from the farm and releases them to the jungle that is full of predator (i.e. p = 50 snakes at the time he/she releases the rabbit). Given the governing equation of the rabbit population is changed to $\frac{dy}{dt} = -p + 3y(1 - \frac{y}{100})$. How long does it take the rabbit population to decrease to half?

5. A brine mixing problem is illustrated in the following figure where a tank contains a liquid of volume $V_0 = 10m^3$ with concentration $C_0 = 0.5\frac{g}{m^3}$ initially and two valves (A & B) are opened simultaneously. The rate of change for the amount of salt in the tank over time is given: $\frac{dx}{dt} = Q_1C_1 - Q_2\frac{x(t)}{V_0 + (Q_1 - Q_2)t}$. Let $x(t)$ = amount of salt; concentration of salt over time= $C(t) = \frac{x(t)}{V(t)}$; $Q_1 = Q_2 = a\frac{m^3}{min}$; $C_1 = 1\frac{g}{m^3}$.

   Replace a with the last three digits of your matric number. For example, if your matric number is KHA110108, your a is thus 108.

   What is the change of the amount of salt and also the change of its concentration over time.?



6. In the lecture note, mathematical modelling of engineering problem such as falling parachutist problem and electrical circuit problem, using differential equation has been demonstrated. Give an example for the mathematical modelling of an engineering problem related to your field of study by using differential equation. Identify the dependent variable, independent variable, parameters and forcing functions for your problem. You are required to present that in the tutorial class. Credit will be given for those who provide example involving calculation or illustration.

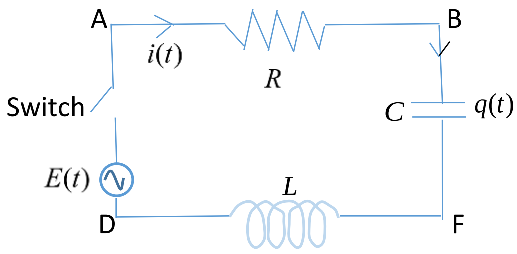
7. Given the governing equation for RLC electrical circuit: $L\frac{d^2q(t)}{dt^2} + R\frac{dq(t)}{dt} + \frac{1}{C}q(t) = E(t)$.

   An inductor of L = 2 henrys and a resistor of R = 10 ohms are connected in series with an emf of E volts. Note that in this case the capacitor has been removed. At t = 0, the switch

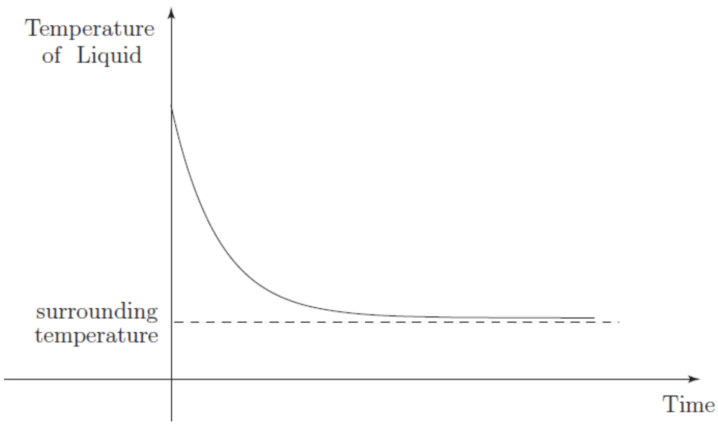
S is closed, thus no charge and current flow at that moment. Find the charge and current at any time $t > 0$ if

a) $E(t) = 40$ volts

b) $E(t) = 20e^{-3t}$ volts



8. When a hot liquid is placed on a cooler environment, experimental observation shows that its temperature decreases to approximately that of its surroundings. A typical graph of the temperature of the liquid plotted against time is shown in the figure below:



After an initially rapid decrease the temperature changes progressively less rapidly and eventually the curve appears to 'flatten out'. Newton's law of cooling states that the rate of cooling of liquid is proportional to the difference between its temperature and the temperature of its environment (the ambient temperature). To convert this into mathematics, let t be the time elapsed (in seconds, s), θ the temperature of the liquid (°C), and θ_0 the temperature of the liquid at the start (t = 0). The temperature of the surroundings is denoted by θ_s. The mathematical expression of Newton's law of cooling is thus $\frac{d\theta}{dt} \propto (\theta - \theta_s)$, or, $\frac{d\theta}{dt} = -k(\theta - \theta_s)$. Here, k is a positive constant of proportionality and the negative sign is present because $(\theta - \theta_s)$ is positive, whereas $\frac{d\theta}{dt}$ must be negative because θ decreases with time. Thus, this justifies the presence of the negative sign.

a) What would be the unit of k?

b) Given the initial condition at $t = 0$ is $\theta = \theta_0$, find the solution to the differential equation $\frac{d\theta}{dt} = -k(\theta - \theta_s)$ that satisfy the initial condition.

c) Plot the graph of θ against t for the solution obtained in b) and indicate θ_0 and θ_s on the axis.

d) If the Newton's law of cooling is written in the form of $\frac{d\theta}{dt} + k\theta = k\theta_s$, solve the differential equation using the integrating factor method. Initial condition is the same as given in b). Compare your answer with the one obtained in b).