Limit and Derrivatives

Limit Laws

Suppose that c is a constant and the limits $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist.

Then,

1. $\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$
2. $\lim_{x\to a}[f(x)-g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$
3. $\lim_{x\to a}[cf(x)] = c\lim_{x\to a} f(x)$
4. $\lim_{x\to a}[f(x)g(x)] = \lim_{x\to a} f(x)\cdot\lim_{x\to a} g(x)$
5. $\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ if $\lim_{x\to a} g(x) \neq 0$

$$\lim_{\theta\to 0}\frac{\theta}{\sin\theta} = 1$$

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$$

Derivatives Rules

Rules	Function	Derivative
Multiplication by constant	cf	cf'
Power Rule	x^n	nx^{n-1}
Sum Rule	$f+g$	$f'+g'$
Difference Rule	$f-g$	$f'-g'$
Product Rule	fg	$fg'+f'g$
Quotient Rule	$\frac{f}{g}$	$\frac{f'g-g'f}{g^2}$
Reciprocal Rule	$\frac{1}{f}$	$-\frac{f'}{f^2}$

Increasing/Decreasing Test

- If $f'(x) > 0$ on an interval, then f is increasing on that interval.
- If $f'(x) < 0$ on an interval, then f is decreasing on that interval.

Derivatives

Function	Function	Derivative
Constant	c	0
Line	x	1
	ax	a
Square	x^2	$2x$
Square Root	$\sqrt{x}$	$\left(\frac{1}{2}\right)x^{-\frac{1}{2}}$
Exponential	e^x	e^x
	a^x	$\ln(a)\,a^x$
Logarithms	$\ln(x)$	$\frac{1}{x}$
	$\log_a(x)$	$\frac{1}{x\ln(a)}$
Trigonometry (x in radians)	$\sin(x)$	$\cos(x)$
	$\cos(x)$	$-\sin(x)$
	$\tan(x)$	$\sec^2(x)$
Inverse Trigonometry	$\sin^{-1}(x)$	$\frac{1}{\sqrt{1-x^2}}$
	$\cos^{-1}(x)$	$-\frac{1}{\sqrt{1-x^2}}$
	$\tan^{-1}(x)$	$\frac{1}{1+x^2}$

Derivatives of Inverse Trigonometric Function

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\csc^{-1}x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$$

Derivatives of Inverse Hyperbolic Function

$$\frac{d}{dx}(\sinh x) = \cosh x \qquad \frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x\coth x$$

$$\frac{d}{dx}(\cosh x) = \sinh x \qquad \frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x\tanh x$$

$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x \qquad \frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$$

Derivatives of Inverse Hyperbolic Function

$$\frac{d}{dx}(\sinh^{-1}x) = \frac{1}{\sqrt{1+x^2}} \qquad \frac{d}{dx}(\operatorname{csch}^{-1}x) = -\frac{1}{|x|\sqrt{x^2+1}}$$

$$\frac{d}{dx}(\cosh^{-1}x) = \frac{1}{\sqrt{x^2-1}} \qquad \frac{d}{dx}(\operatorname{sech}^{-1}x) = -\frac{1}{x\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tanh^{-1}x) = \frac{1}{1-x^2} \qquad \frac{d}{dx}(\coth^{-1}x) = \frac{1}{1-x^2}$$

Concavity Test

- If $f''(x) > 0$ for all x in I, then the graph of f is concave upward on I.
- If $f''(x) < 0$ for all x in I, then the graph of f is concave downward on I.

The First Derivative Test

Suppose that c is a critical number of a continuous function f.

- If f' changes from positive to negative at c, then f has a local maximum at c.
- If f' changes from negative to positive at c, then f has a local minimum at c.
- If f' does not change sign at c, then f has no local maximum or minimum at c.

The Second Derivative Test

Suppose f'' is continuous near c.

- If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.
- If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.

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